**ATT IPP Pre-work:**

Estimated Time Needed to Complete: 45-60 minutes

Directions:

Complete all IPP pre-work to the best of your ability. Pre-work includes:

* Read the entire protocol (in a separate pre-work document) and be able to articulate the steps of the protocol
* Read phase 0 for an Exercise based lesson and a Task based lesson (below) and come prepared to ask questions about effective planning for Phase 0
* Do each problem using 2-3 strategies and/or representations; Please scan and email your representations/strategies and solutions to your course facilitator by 8/2 (NY) or 8/9 (CT)

Facilitators:

* NY
  + 5th: Bridget Venter
  + 6th: Marcella Chibbaro
  + 7th: Jenna Casano
  + 8th: Josh Alfred
* CT
  + 5th: Andrew Navratil
  + 6th: Carla Seeger
  + 7th: Wooly Pierre
  + 8th: Hannah Laplante

Pre-work:

* [5th Grade Exercise Based Problem](#Gr5EBL)
* [5th Grade Task Based Problem](#Gr5TBL)
* [6th Grade Exercise Based Problem](#Gr6EBL)
* [6th Grade Task Based Problem](#Gr6TBL)
* [7th Grade Exercise Based Problem](#Gr7EBL)
* [7th Grade Task Based Problem](#Gr7TBL)
* [8th Grade Exercise Based Problem](#Gr8EBL)
* [8th Grade Task Based Problem](#Gr8TBL)

Criteria for selecting an exercise:

* Draws thinking towards mathematics to be used and learned; is relatively narrowly focused on a strategy, concept or skill
* May be difficult or easy, complex or simple, but never puzzling
* The path(s) towards the solution is(are) often apparent
* Incorporates the following Key Cognitive Strategies (Conley):
  + Problem Formation: requires planning and use of reasoning skills
  + Research: lends itself to strategic selection and use of tools
  + Interpretation: requires planning and use of reasoning skills; requires understanding, identification and/or application of one or more concepts and skills
  + Communication: requires students to demonstrate evidence of their thinking, fluency and conceptual understanding through use of models, work shown and/or written explanations; requires evidence to be provided and may require development of logical argument for concepts or steps
  + Precision & Accuracy: requires attention to appropriate rules of precision when tending to work in written, oral, or symbolic form

Criteria for selecting a task:

* Draws thinking towards mathematics to be used and learned
* Incorporates the following Key Cognitive Strategies (Conley):
  + Problem Formation: requires planning and use of reasoning skills
  + Research: lends itself to strategic selection and use of tools
  + Interpretation: requires planning and use of reasoning skills; requires understanding, identification and/or application of multiple concepts and skills
  + Communication: requires students to demonstrate evidence of their thinking, fluency and conceptual understanding through use of models, work shown and/or written explanations; requires evidence to be provided and may require development of logical argument for concepts
  + Precision & Accuracy: requires attention to appropriate rules of precision when tending to work in written, oral, or symbolic form
* Non-routine and complex
* Solution path is neither stated or obvious; may be multiple solution paths; may be multiple solutions

***5th Grade Exercise Based Lesson Phase 0***

Lesson 1

Aim: SWBAT explore volume by building 3D figures and counting with unit cubes

Content Standards:

**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

1. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
2. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.

Practice Standards:

* SMP3 – Construct viable arguments and critique the reasoning of others
* SMP4 – Model with mathematics
* SMP5 – Use appropriate tools strategically

Key Points:

* Why
* Understanding and applying properties of 2D and 3D figures allows us to qualitatively and quantitatively describe and measure attributes of figures.
* What
* The properties of 3D figures can be used to solve problems
* The properties of 3D figures can be used to describe and identify the type of solid; conversely, knowing the type of 3D figure allows one to describe the solid’s properties. Students will focus on Cubes and Rectangular prisms, but will be expected to ID properties of other figures given a picture or model.
* Introduce important vocabulary: solid, volume, unit cube, base, face, edge, vertex, (rectangular) prism, cube
* How
* Building solids: Given the volume in cubic centimeters, students will collect the appropriate number of unit cubes and build figures with a given volume.

*\*\*There are other KPs that apply to this lesson; however, I’ve only included the ones relevant to the selected exercise. Please see the lesson plan on Better Lesson for the entire plan.*

Problem: Students are asked to first to draw a square with an area of 4 square cm using centimeter grid paper. Then, students are asked to place three layers of 4 centimeter unit cubes to form a rectangular prism. Finally, students are prompted to determine the total number of cubes used to create the prism using a variety of strategies.

*\*\*While this does not seem like a particularly rigorous exercise, this is their first exposure to volume and is laying the groundwork for understanding the two formulas that can be used for calculating volume of a rectangular prism.*

***5th Grade Task Based Lesson – Phase 0***

Lesson 5

Learning Goals: Develop strategic competence, adaptive reasoning, and problem solving skills by:

* Solving real world problems involving volume
* Organizing thinking and work in a logical way given the application of volume and constraints described in the task
* Thinking flexibly when working with formulas
* Apply understanding of volume to draw and explain conclusions

Content Standards:

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

1. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole number products as volumes, e.g., to represent the associative property of multiplication.
2. Apply the formulas *V = l x w x h* and *V = b x h* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
3. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Practice Standards:

* SMP1: Make sense of problems and persevere in solving them
* SMP2: Reason abstractly and quantitatively
* SMP4: Model with mathematics

Key Points:

* Why:
* Understanding and applying properties of 2D and 3D figures allows us to qualitatively and quantitatively describe and measure attributes of figures.
* Coming up with multiple strategies often helps us develop more efficient methods.
* What:
* Volume measures the amount of space inside a 3D figure, measured in cubic units
* Figures with the same volume can have different dimensions
* A figure with a given volume can only be a cube if the volume is equivalent to the side length cubed
* An organized list is an efficient way to determine a number of different combinations
* Finding the factors of a number is an efficient strategy that can be used to determine the dimensions of a rectangular prism given the volume of the figure because the volume of a rectangular prism is a product of three factors
* How: General How KPs for solving novel problems (*more specific strategies are intentionally left off and should be identified during the protocol)*
* Understand the problem using appropriate annotations
* Represent the known and unknown information
* Seek a strategy to solve the problem by applying understanding of mathematical connections and rules
* Determine a solution and check the reasonableness of the answer using alternate methods and re-contextualizing the solution

Problem:

Cari is the lead architect for the city’s new aquarium. All of the tanks in the aquarium will be rectangular prisms where the side lengths are whole numbers.

* 1. Cari knows that a certain species of fish needs at least 80 cubic feet of water in their tank.
     1. Create 3 separate tanks that hold exactly 80 cubic feet of water with different sets of numerical values used for all three dimensions.
     2. Create 3 separate tanks that hold exactly 80 cubic feet of water using the same three numerical values in different order. Describe what is different about each tank.
     3. Can you create a tank that holds 80 cubic feet and has the same whole number measurement for the length, width and height? Explain.

\*\**Including part B for you reference. No need to solve as it is part of the Extension and not the initial Exploration*

* 1. In the back room of the aquarium, Cari realizes that the ceiling is only 6 ft. high, and the floor is 12 ft. by 12 ft. She needs to create a tank that can hold exactly 100 cubic feet of water, has whole number measurements, and fits in the room. How many different ways could she build a tank that will fit in the room? Prove your answer.

***6th Grade Exercise Based Lesson – Phase 0***

Lesson 17

Aim: SWBAT divide a decimal number by a decimal number by multiplying the divisor and dividend by a multiple of ten in order to create an equivalent expression with a whole number divisor; SWBAT explain and apply the algorithm for division of decimals.

Content Standards:

6.NS.3 Fluently, add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation

Practice Standards:

* SMP1: Make sense of problems and persevere in solving them
* SMP6: Attend to precision

Key Points:

* Why:
  + Deep conceptual understanding of operations leads to fluency
* What:
  + We rewrite a division expression with decimal in the divisor using an equivalent expression with a whole number divisor to facilitate division.
  + Multiplying the divisor and dividend in a division expression by the same value results in an equivalent expression because of the identity property of multiplication.
  + When dividing by a divisor >1, the quotient will be less than the dividend because the number of groups of the divisor cannot exceed the value of the dividend since each group size is greater than 1; when dividing by a divisor <1, the quotient will be greater than the dividend because the number of groups of the divisor can exceed the value of the dividend since each group size is less than 1.
* How:
  + Write the division expression using fraction notation
  + Multiply the dividend and divisor by a power of ten large enough to write an equivalent expression with a whole number divisor
  + Divide using the standard algorithm
  + Check using estimation or inverse operations

Problem:Jerod is making candles from beeswax. He has 4.8 pounds of beeswax. Each candle uses 0.4 pound of beeswax. Jerod thinks that he can make 1.2 candles and his sister thinks he can make 12 candles. Who is correct? How do you know?

***6th Grade Task Based Lesson – Phase 0***

Lesson 1

Learning Goals:

* Develop Conceptual Understanding by:
  + Applying understanding of factors to develop conceptual understanding of greatest common factor, and understanding how and why the GCF represents the desired solution given their understanding of the concept and problem context.
* Develop strategic competence, adaptive reasoning, and problem solving skills by:
  + Representing and solving real world problems involving factors and the GCF
  + Organizing thinking and work in a logical way given the application of factors and the GCF, and constraints described in the task
  + Thinking flexibly when working with factors
  + Apply understanding of factors to draw and explain conclusions

Content Standards:

**6.NS.4** SWBAT determine all the factors of a number less than or equal to 100 and the GCF of two numbers less than or equal to 100.

Practice Standards:

* [SMP1](http://www.corestandards.org/Math/Practice/MP1/) Make sense of problems and persevere in solving them.
* [SMP2](http://www.corestandards.org/Math/Practice/MP2/) Reason abstractly and quantitatively.
* [SMP3](http://www.corestandards.org/Math/Practice/MP3/) Construct viable arguments and critique the reasoning of others.
* [SMP4](http://www.corestandards.org/Math/Practice/MP4/) Model with mathematics.

Key Points:

* Why:
* Understanding when a math concept is applicable to real world contexts allows one to solve common place problems more efficiently.
* What:
* The GCF represents the largest common number by which two or more numbers are divisible.
* An organized list is an efficient way to determine a number of different combinations
* Drawing a picture helps visualize the context of a problem
* Introduce important vocabulary: Factor, Factor pair, Divisible, Common Factors, and Greatest Common Factor
* How: General How KPs for solving novel problems (*more specific strategies are intentionally left off and should be identified during the protocol)*
* Understand the problem using appropriate annotations
* Represent the known and unknown information
* Seek a strategy to solve the problem by applying understanding of mathematical connections and rules
* Determine a solution and check the reasonableness of the answer using alternate methods and re-contextualizing the solution

Problem:

Jasmine is making identical balloon arrangements for a party. She has 32 maroon balloons, 24 white balloons, and 16 orange balloons. She wants each arrangement to have the same number of each color. What is the greatest number of arrangements that she can make if every balloon is used?

***7th Grade Exercise Based Lesson – Phase 0***

Lesson 1

Aim:

SWBAT represent and begin to develop an understanding of addition of integers by using a horizontal or vertical number line

* 1. SWBAT understand p + q as the number located a distance |q| from p in a positive or negative direction depending on the sign of q.
  2. SWBAT show and understand that an integer and its opposite have a sum of 0 (are additive inverses)

Content Standards: 7.NS.1

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

1. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
2. Understand *p + q* as the number located a distance |*q*| from p, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
3. Understand subtraction of rational numbers as adding the additive inverse, *p – q = p + (-q)*. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
4. Apply properties of operations as strategies to add and subtract rational numbers.

Practice Standards:

* SMP2: Reason abstractly and quantitatively
* SMP4: Model with Mathematics
* SMP5: Use appropriate tools strategically

Key Points:

* Why:
  + Deep conceptual understanding of operations leads to fluency
* What:
  + We can represent the addition of positive and negative integers using a vertical or horizontal number line; the context of the problem determines the orientation of the number line.
  + In the expression p + q, p represents the starting point and q represents the distance moved from p in either the negative or positive direction depending on the sign of q.
* How:
  + To represent the sum of two integers, p and q…
* Begin by drawing a number line and labeling the intervals. Choose an appropriate range and interval for the expression you are modeling.
* Plot and label the first addend as p on the number line and plot the point.
* Indicate the direction (positive or negative) with an arrow.
* The q value indicates how many units long with arrow will be. Label the arrow with q.
* Plot the sum as the second point on the number line and label it as the sum.
* Write an equation to represent the calculation performed

Problem: The table below shows the temperature changes Monday morning in Bedford, NY over a five-hour period. The beginning temperature at 5:00 AM was -13°F. What was the temperature at 9 AM?

|  |  |
| --- | --- |
| **Change in Temperature** | |
| 5:00 AM – 6:00 AM | -3°F |
| 6:00 AM – 7:00 AM | +8°F |
| 7:00 AM – 8:00 AM | +5°F |
| 8:00 AM – 9:00 AM | -5°F |

*\*\*Note: While this problem could be solved by simply drawing a number line and going step-by-step, the numbers are intentionally set up to also being to establish alternate strategies, i.e. find the total positive change and total negative change first; notice that the total positive change and starting temperature result in a combined temperature of 0°F; or, only add the first two temperature changes since the last two have a 0 sum. Additionally, there is great cause to discuss the commutative property.*

***7th Grade Task Based Lesson – Phase 0***

Lesson 4

Learning Goals: SWBAT calculate and interpret sums of integers in real-world contexts

* Develop and demonstrate Procedural Fluency by accurately and efficiently adding integers
* Develop strategic competence, adaptive reasoning, a product disposition, and problem solving skills by:
  + Trying multiple approaches to adapt to the constraints of the problem and find an appropriate solution
  + Persevering despite experiencing failure
  + Reflecting on previous attempts/strategies and adjusting course given the outcomes

Content Standards: 7.NS.1

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

1. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
2. Understand *p + q* as the number located a distance |*q*| from p, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
3. Understand subtraction of rational numbers as adding the additive inverse, *p – q = p + (-q)*. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
4. Apply properties of operations as strategies to add and subtract rational numbers.

Practice Standards:

* SMP1: Make sense of problems and persevere in solving them
* SMP3: Construct viable arguments and critique the reasoning of others
* SMP4: Model with Mathematics

Key Points:

* Why:
  + Operations with rational numbers are prevalent in our world
  + Fluency with operations leads to greater flexibility for thinking about mathematical and real world applications
* What:
  + We can add positive and negative integers using multiple methods – number line, chips, mental math, or computation.
  + Opposite quantities combine to make 0
  + Two negative integers combine to be a negative with a lower value; two positive integers combine to be a positive integer with a higher value; a negative and a positive integer combine to be a greater or lesser positive or negative value depending on the magnitudes of the integers being combined.
* How: General How KPs for solving novel problems (*more specific strategies are intentionally left off and should be identified during the protocol)*
* Understand the problem using appropriate annotations
* Represent the known and unknown information
* Seek a strategy to solve the problem by applying understanding of mathematical connections and rules
* Determine a solution and check the reasonableness of the answer using alternate methods and re-contextualizing the solution

Problem: Jorge is playing a game involving one die, labeled 1-6, with his brother. Here are the rules:

* Even numbers have negative point values
* Odd numbers have positive point values

For example, Jorge rolled a 4. The 4 is worth -4 points. His brother rolled a 3, which is worth +3 points.

Jorge rolled the die four times resulting in a total of 0 points.

1. What are three combinations of dice rolls he could have thrown? Write an equation to represent the combinations you came up with that result in a sum of 0 points.
2. Describe a rule that can be applied to any set of four rolls to result in a sum of 0.
3. Can you ever roll two dice and have a sum of 0? Why? If not, how would you change the rules of the game to make it possible?

***8th Grade Exercise Based Lesson – Phase 0***

Lesson 1

Aim: SWBAT solve linear equations in one variable and determine when a linear equation has one solution, no solution or infinitely many solutions.

Content Standards:

Solve linear equations in one variable.

1. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form or results (where and are different numbers).
2. Solve linear equations with rational coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.

Practice Standards:

* SMP3: Construct viable arguments and critique the reasoning of others
* SMP7: Look for and make use of structure
* SMP8: Look for and express regularity in repeated reasoning

Key Points:

* Why:
  + Understanding and looking for structures within mathematical representations allows one to reason and draw conclusions about the representation.
* What:
  + An equation is a mathematical statement that two expressions are equal.
  + One way to determine the number of solutions an equation has is by simplifying the equation. One can also deduce the number of solutions through observation of structure prior to the simplest form of the equation.
  + If an equation simplifies to the form x=a, there is one solution.
  + If an equation simplifies to the form a=a, there are infinitely many solutions (this is called an identity).
  + If an equation simplifies to the form a=b, there is no solution (this is called a contradiction).
* How:
* Distribute and/or combine like terms if necessary.
* Move all variables to one side of the equation using inverse operations if necessary.
* Isolate the variable using inverse operations.
* Check your answer using substitution.
  + Inspect the simplest form of the equation to determine number of solutions

Problem: Which equation(s) has no solutions?

1. 6x + 4 = 6x + 1
2. 3(x – 2) = 2x – 6 + x
3. 12x + 5 – 6x = 8x + 3
4. 2x – 12 + 6x = 2x(1 + 3) + 12

How do you know?

***8th Grade Task Based Lesson – Phase 0***

Lesson 7

Learning Goals:

* Develop conceptual understanding by using appropriate tools to identify angle measurements and draw conclusions about angle relationships.
* Develop strategic competence and adaptive reasoning by solving mathematical problems and forming conclusions about angle relationships with justification.

Content Standards:

Use informational arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, ~~and the angle-angle criterion for similarity of triangles.~~ *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Practice Standards:

* SMP3: Construct viable arguments and critique the reasoning of others
* SMP5: Use appropriate tools strategically
* SMP7: Look for and make use of structures
* SMP8: Look for and express regularity in repeated reasoning

Key Points:

* Why: Understanding mathematical rules deeply helps develop new understandings or helps one solve more demanding problems by applying the known rules.
* What:
  + When two parallel lines are cut by a transversal, specific angle relationships form between the angles created by the two points of intersection.
  + Relevant vocabulary: Congruent, Transversal, Parallel Lines, Corresponding Angles, Alternate Interior Angles, Alternate Exterior Angles.
* How:

With protractor:

* + Create a transversal
  + Using a protractor, measure all angles created by lines at points of intersection
  + Identify congruent angle measurements

Without protractor:

* + Identify congruent angles using known angle relationships
  + Determine measurement of unknown angles using known measurements and relationships between angles

Problem:

Below is a set of parallel lines. Using your ruler, draw a transversal – a line that goes through both of the parallel lines.



The top angle on the left should be named A, the top angle on the right should be named B, and so forth. It should look like the example below (but the line cutting the parallel lines can cut them at any angle.

a

b

c

d

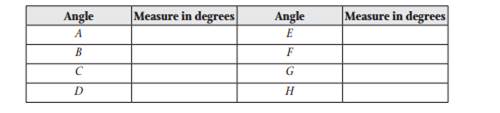
e

f

g

h

Measure your angles and record them in the table below.



What angles are congruent?

What pairs of angles are supplementary?

Below is a set of non-parallel lines. Using your ruler, draw a transversal – a line that goes through both of the lines.

The top angle on the left should be named A, the top angle on the right should be named B, and so forth. It should look like the example below (but the line cutting the parallel lines can cut them at any angle.

a

b

c

d

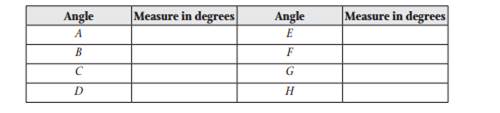
e

f

g

h

Measure your angles and record them in the table below.



What angles are congruent?

What pairs of angles are supplementary?

What do you notice about the angle relationships formed when two lines that are parallel and are not parallel are intersected by a transversal? Are there any rules or relationships that are consistent? If so, what are they? Why do you think they are true? What is different between the angle relationships formed? Why do you think those differences exist?